Nuclear Structure from Gamma-Ray Spectroscopy

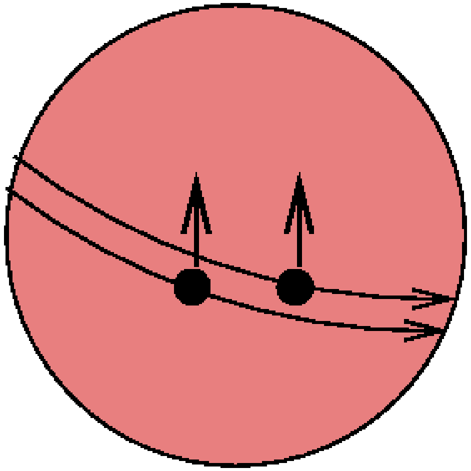
2019 Postgraduate Lectures

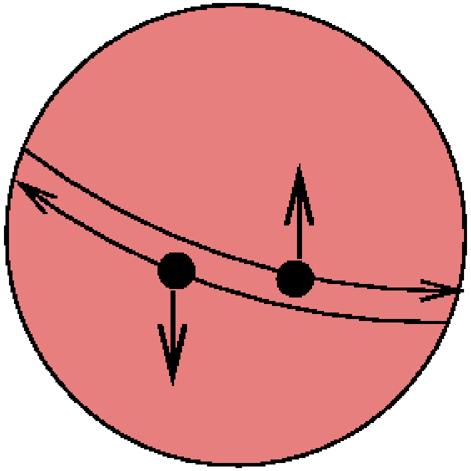
Lecture 5: Pairing & Quasiparticles

Pairing: Experimental Evidence

- The ground states of all even-even nuclei have $I^\pi = 0^+$
- The binding energies of odd-even nuclei are less than the mean value of the two neighbouring even-even nuclei
- Doubly odd nuclei are even less bound
- Nuclear moments of inertia are only 30-50% of the rigid-body value at low spin

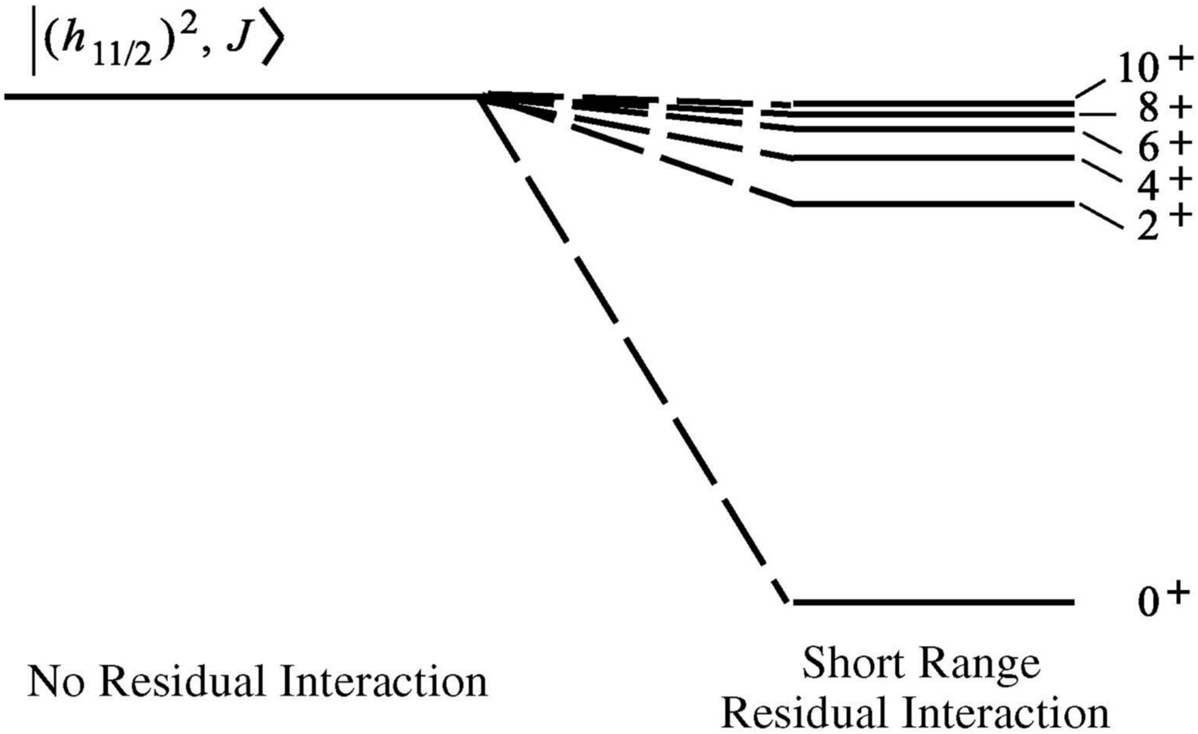
Time Reversed Orbits





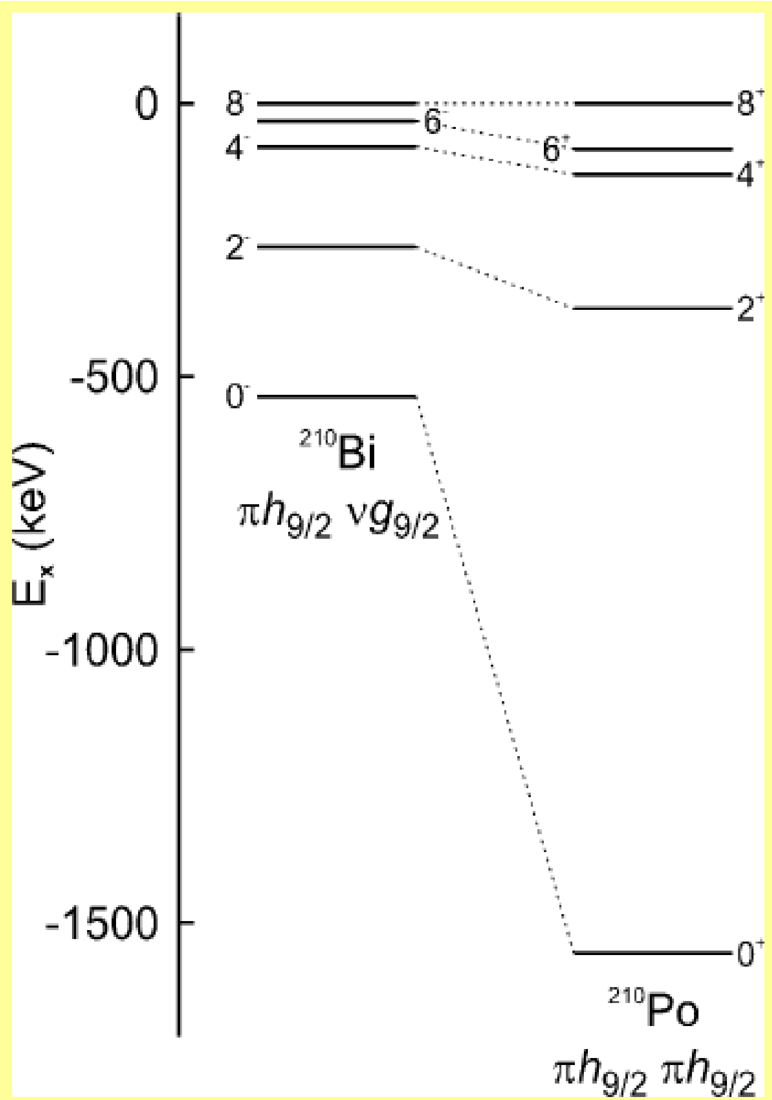
- The greatest overlap would occur if two particles could orbit in the same level
- Not allowed (PEP) !
- The next greatest overlap occurs for particles in 'time reversed' orbits
- The spins cancel to give $I^{\pi} = 0^{+}$

Coupling Two Particles



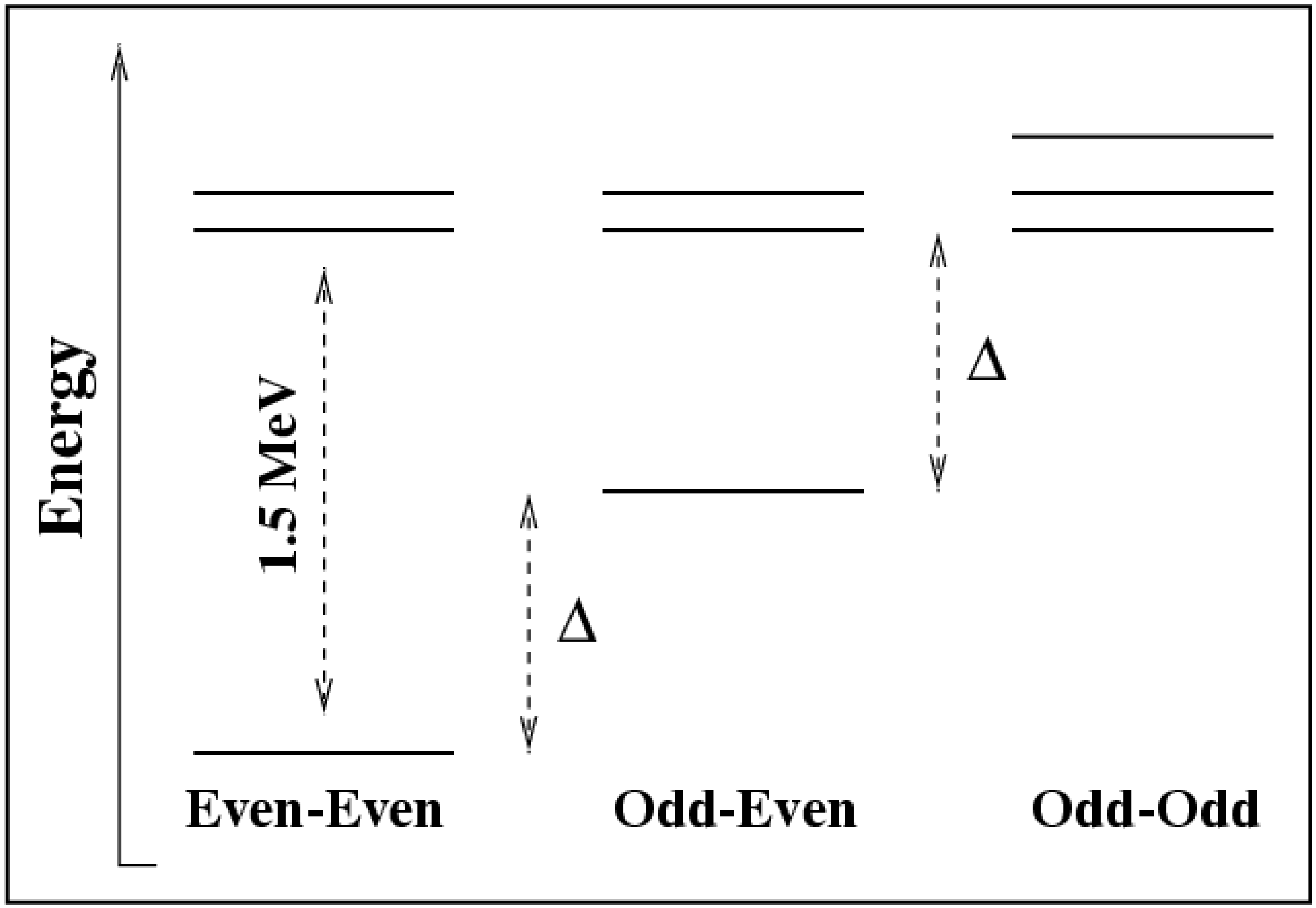
- The short-range (pairing) residual interaction yields an energetically favoured 0^+ state

Favoured 0^+ Ground State



- In ^{210}Po the configuration outside the doubly closed shell core of ^{208}Pb is $(\pi h_{9/2})^2$.
- If there were no interaction between these two protons, i.e. if they behaved like independent particles, the various $(h_{9/2})^2$ spin couplings, which reflect the orbital alignments, would lead to states degenerate in energy.
- Correlated pair of two protons
- Energy gain ≈ 2Δ

Odd-Even Mass Difference



Pairing Energies

- The neutron separation energy is:

$$S_n = B(A,Z) - B(A-1,Z) = M(A-1,Z) - M(A,Z) + M_n$$

where $B(A,Z)$ in the nuclear binding energy

- The proton separation energy is:

$$S_p = B(A,Z) - B(A-1, Z-1) = M(A-1,Z-1) - M(A,Z) + M_H$$

- The pairing energies are:

$$P_n(A,Z) = S_n(A,Z) - S_n(A-1,Z) \quad \text{(neutron)}$$

$$P_p(A,Z) = S_p(A,Z) - S_p(A-1,Z-1) \quad \text{(proton)}$$

Pairing Hamiltonian

- The Hamiltonian including a two-body monopole (i.e. $I = 0$) pairing interaction is:

$$H = H_{sp} + H_{pair} = \sum \epsilon_u [a_u^\dagger a_u + a_{\bar{u}}^\dagger a_{\bar{u}}] - G \sum a_{u1}^\dagger a_{\bar{u}1}^\dagger a_{\bar{u}2} a_{u2}$$

- Here $a^\dagger$ and a are particle creation and annihilation operators

- The first term is the sum of single-particle energies

- The second term contains the pairing interaction that annihilates a pair of particles in time reversed orbits $|u_2\rangle$ and $|\bar{u}_2\rangle$ and simultaneously creates a pair in time reversed orbits $|u_1\rangle$ and $|\bar{u}_1\rangle$

Chemical Potential λ

- The energy increase of the condensate per particle added defines the chemical potential λ

- The Hamiltonian is: $H' = H - \lambda\tilde{N} = H_{sp} + H_{pair} - \lambda\tilde{N}$

  where $\tilde{N}$ is the particle number operator

- The two-body monopole pairing interaction is:

$$H_{pair} = -\tfrac{1}{4}G\, P^{\dagger} P$$

where the pair creation and annihilation operators are:

$$P^{\dagger} = \sum a_{u}^{\dagger}\, a^{\dagger}_{\bar{u}} \quad \text{and} \quad P = \sum a_{\bar{u}}\, a_{u}$$

Pairing Strength G

- The strength of the pairing term G is a positive constant
- It is larger for high-j orbitals and depends on the spatial overlap of the two nucleons
- The strength decreases with mass since in heavier nuclei the outer nucleons are further apart
- The strength is also lower for protons than neutrons because of Coulomb repulsion
- Approximately:

$$G_p = 17/A \text{ MeV} \text{ and } G_n = 23/A \text{ MeV}$$

Pairing Gap Δ

- The pairing term contains the product of two creation and two annihilation operators

- In order to simplify the calculations, the term $P^\dagger P$ (product) is replaced by $P^\dagger + P$ (sum) and:

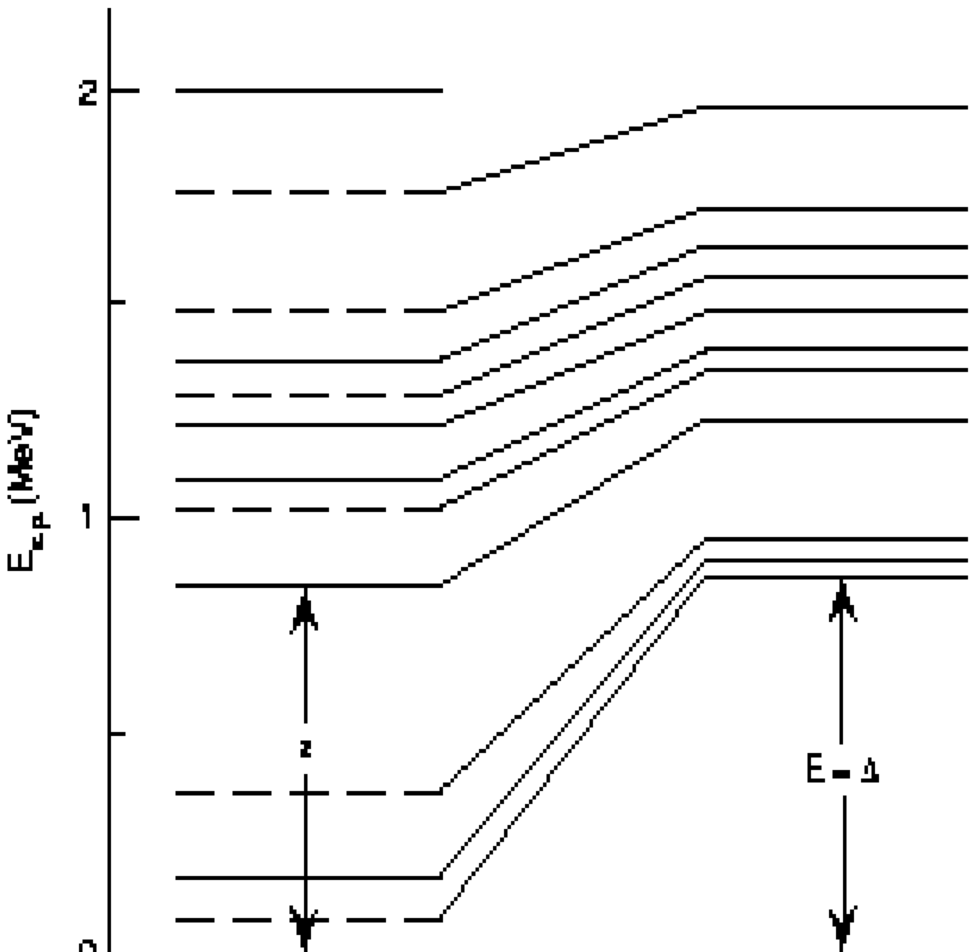
$$H_{pair} = -\tfrac{1}{2}\Delta\,[P^\dagger + P]$$

which introduces the pairing gap parameter Δ

- Particle number is now not conserved ! The chemical potential λ is now treated as a Lagrange multiplier and is varied to produce the correct particle number:

$$\langle \Psi | \tilde{N} | \Psi \rangle = N \qquad \lambda = -\,\partial E / \partial N$$

Single Particle Levels with Pairing



- An energy gap between the ground state and first excited state of ~ Δ opens up
- The excited states become bunched together
- A rough estimate of the energy required to create a particle-hole excitation is 2Δ

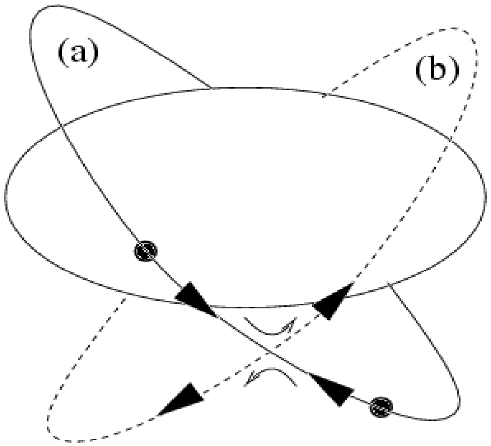
Nuclear Ground State

- Nuclei in their ground states are in specific configurations: some pairs of nucleons are above the Fermi surface (λ) and some states below the Fermi surface are empty
- With pairing, states are not always full or always empty but filled for part of the time or empty for part of the time
- The probability of a given level ϵ_u being occupied by a particle is:

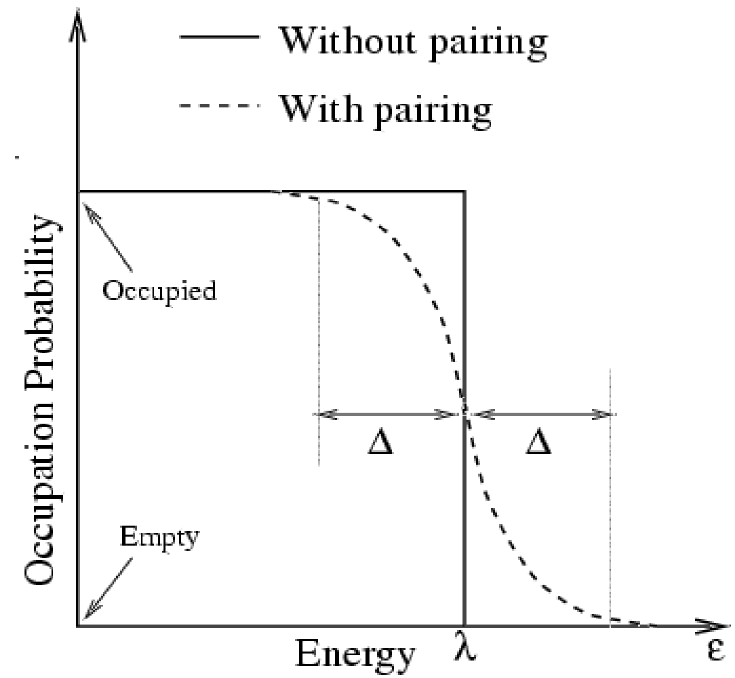
$$P_u(\epsilon_u) = \tfrac{1}{2}\{\, 1 + (\epsilon_u - \lambda) \,/\, \sqrt{[\, (\epsilon_u - \lambda)^2 + \Delta^2 \,]} \,\}$$

- Now $P_u(\epsilon_u) \neq 0$ or 1 around the Fermi surface !

Scattering Between Orbits



- Pairs of particles scatter from one orbit to another, induced by the pairing interaction
- The particles change orbits in pairs so $I^{\pi} = 0^{+}$



- Since the orbits have different energies, the Fermi surface is smeared out over a region ±Δ (±1.5 MeV)

Quasiparticles

- A further simplification is to replace the pairwise interacting particles by a gas of noninteracting 'quasiparticles', whose energies are then simply additive

- A quasiparticle may be considered as a mixture of a particle and hole states

- The Bogoliubov-Valatin transformation changes the particle basis $(a^{\dagger},a)$ into the quasiparticle basis $(\alpha^{\dagger},\alpha)$:

$$\alpha_u^{\dagger} = U_u a_u^{\dagger} + V_u a_{\bar{u}} \quad ; \quad a_u^{\dagger} = U_u \alpha_u^{\dagger} - V_u \alpha_{\bar{u}}$$

$$\alpha_{\bar{u}}^{\dagger} = U_u a_{\bar{u}}^{\dagger} - V_u a_u \quad ; \quad a_{\bar{u}}^{\dagger} = U_u \alpha_{\bar{u}}^{\dagger} + V_u \alpha_u$$

The Quasiparticle Vacuum

- The transformation coefficients U_u and V_u can be obtained following a BCS treatment (superconductivity)

- The BCS wavefunction is of the form:

$$|\Psi_{BCS}\rangle = \Pi_u [U_u + V_u a_u^\dagger a_{\bar{u}}^\dagger] |0\rangle$$

where $|0\rangle$ denotes the vacuum state of the particles and $|\Psi_{BCS}\rangle$ represents the quasiparticle vacuum

- U_u and V_u represent occupation amplitudes ('empty' and 'filled', respectively) and hence:

$$|U_u|^2 + |V_u|^2 = 1$$

Quasiparticle Energies

- Expressions for U_u and V_u are:

$$U_u = (1/\sqrt{2}) \{ 1 + (\epsilon_u + \lambda) / E_u \}^{1/2}$$

$$V_u = (1/\sqrt{2}) \{ 1 + (\epsilon_u - \lambda) / E_u \}^{1/2}$$

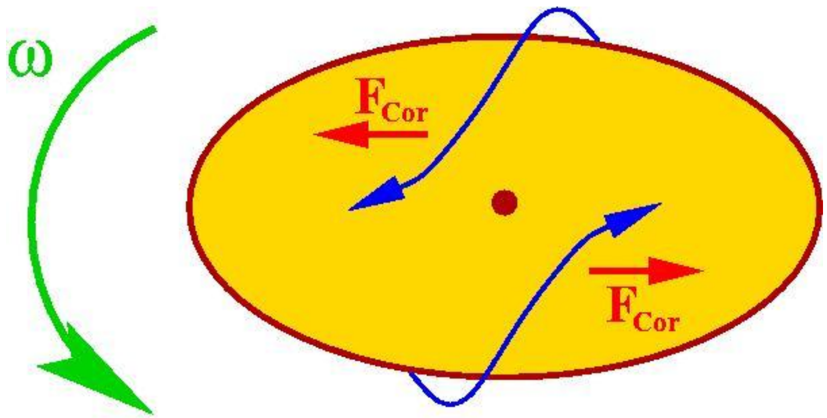
- The quasiparticle energy of a state $|u\rangle$ relative to the ground state is:

$$E_u = \sqrt{[(\epsilon_u - \lambda)^2 + \Delta^2]}$$

where ϵ_u is the single-particle energy. Note that the lowest excited state of a paired nucleus is at a higher energy than for the unpaired case

- The pair gap parameter may be expressed: $\Delta = G\sum U_u V_u$

Destruction of Pairing



- Strong external influences may destroy the superfluid nature of the nucleus

- In the case of a superconductor, a strong magnetic field can destroy the superconductivity: the 'Meissner Effect'

- For the nucleus, the analogous role of the magnetic field is played by the Coriolis force, which at high spin, tends to decouple pairs from spin zero and thus destroy the superfluid pairing correlations

Coriolis Antipairing Effects

- Classically the Coriolis force is given by:

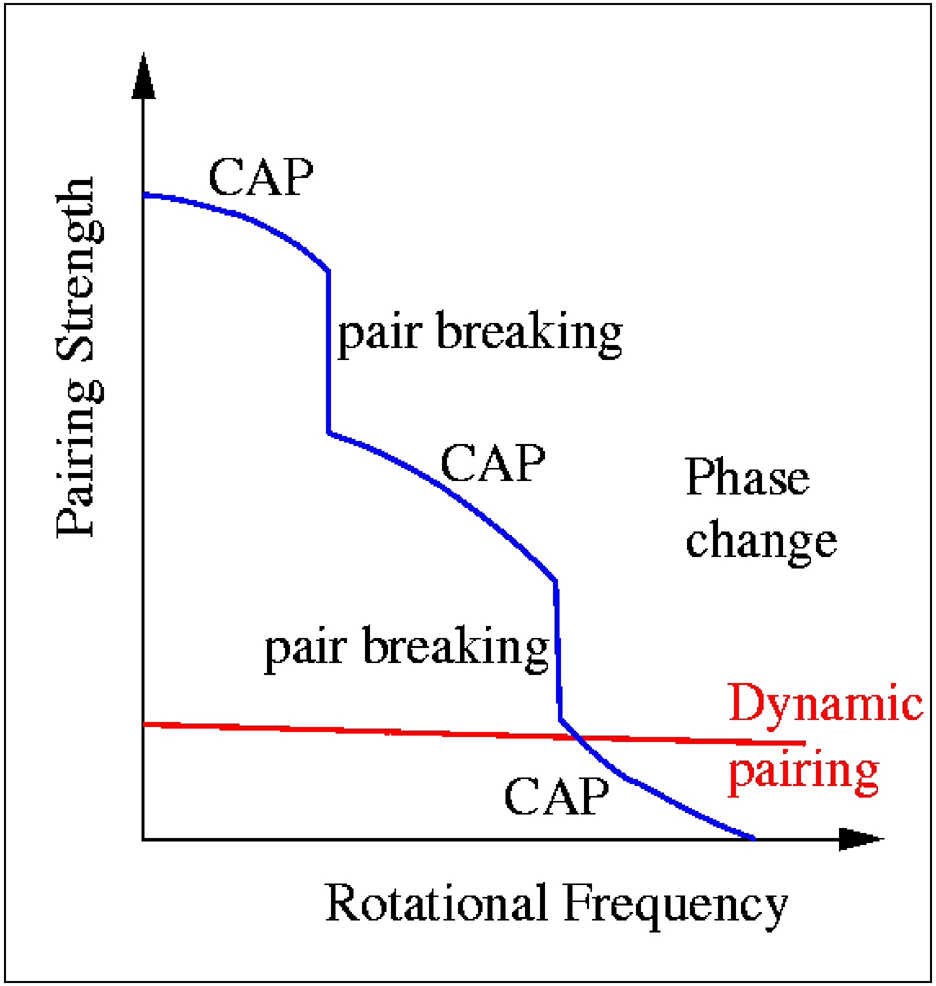
$$\underline{F}_{Cor} = -2m\,[\underline{\omega} \times \underline{v}]$$

- Coriolis Antipairing (CAP): the magnitude of Δ gradually and smoothly decreases and the nuclear moment of inertia ($\propto \omega^2$) increases

- Rotational Alignment: At spin ~ $12\hbar$, the Coriolis force is strong enough to break a specific pair of valence nucleons and align their individual angular momenta along the rotation axis

- High-j low-Ω particles are the most susceptible

Demise Of Pairing



- CAP and rotational alignments diminish the magnitude of the nuclear pairing
- Eventually the nucleus may enter an unpaired phase at high spin
- In addition to static pairing, dynamic pairing correlations occur resulting from fluctuations in Δ

Backbending

- The breaking of a specific nucleonic pair and the rotational alignment of the angular momenta leads to a characteristic 'S' shape of the nuclear Spin vs Frequency

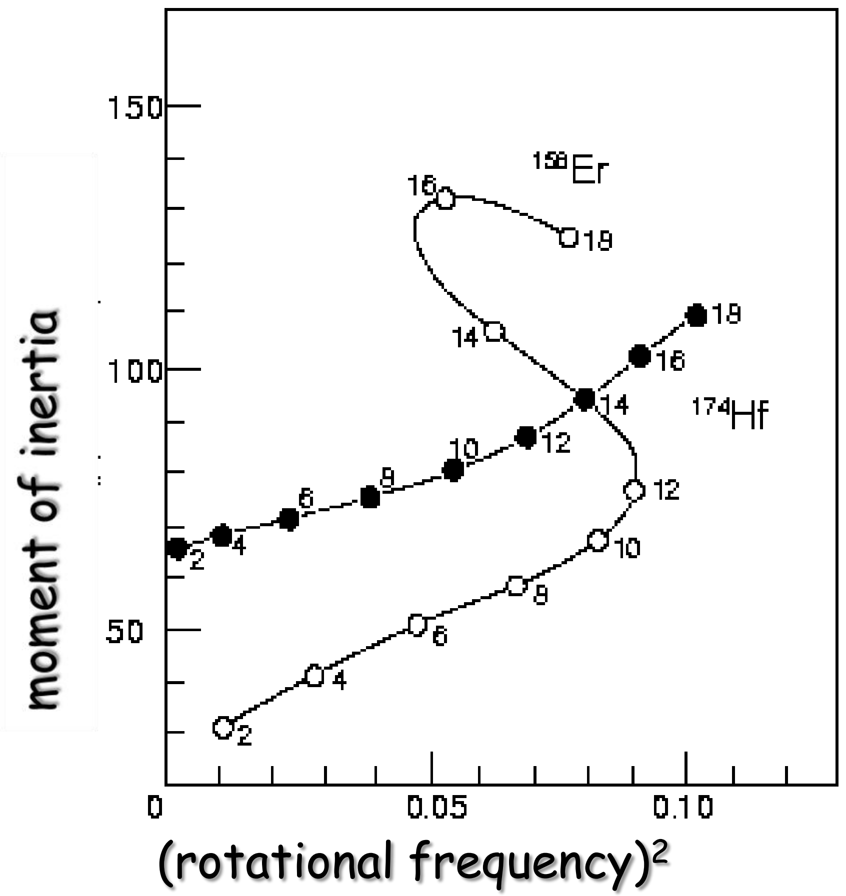
- A contribution of the nuclear spin now comes from single particles:

$$I = R + J$$

with $J \approx (j_{x,max} + j_{x,max} - 1)$ in accordance with the PEP

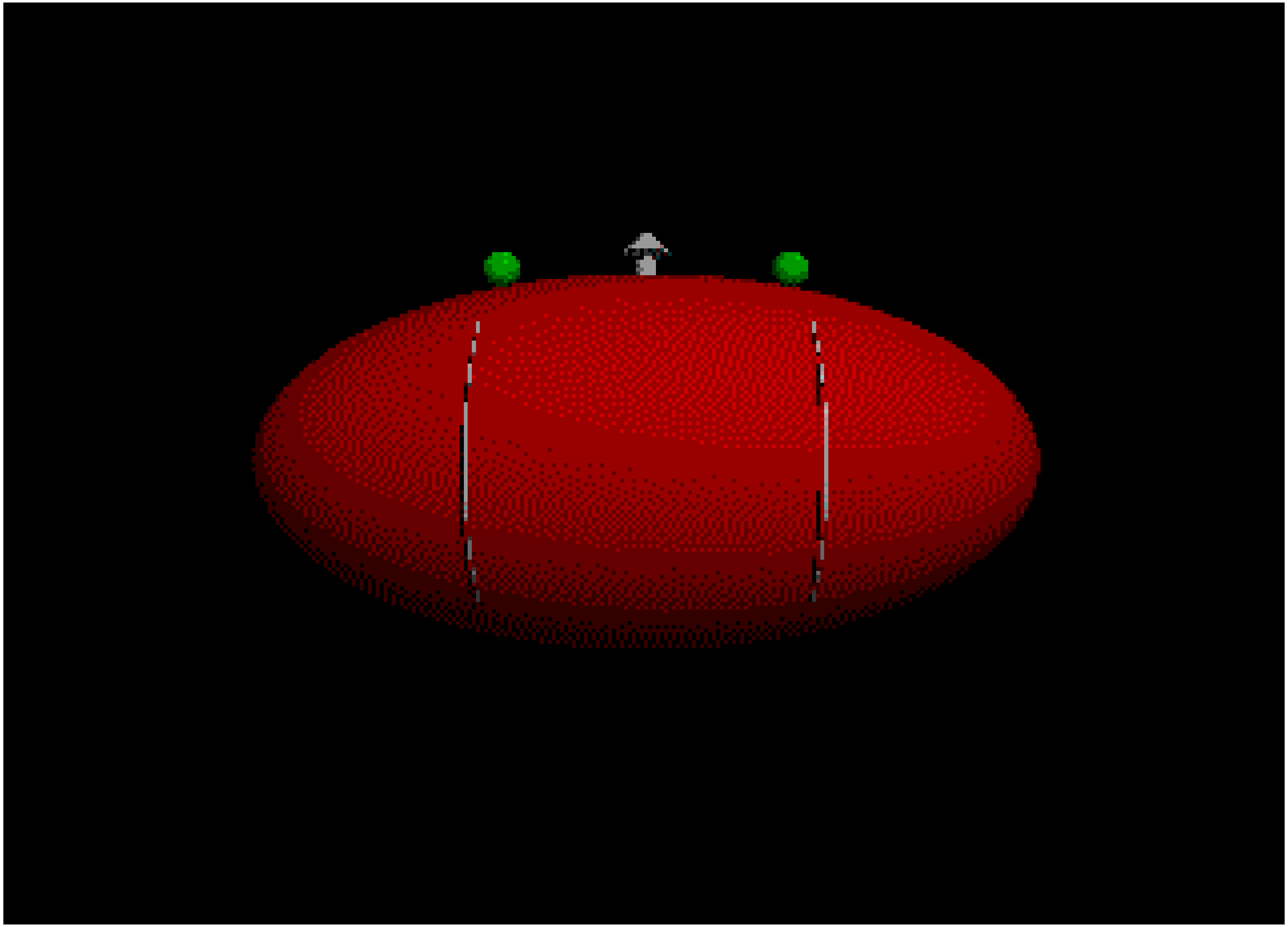
- The nucleus 'changes gear', i.e. slows down while maintaining the angular momentum

Backbending

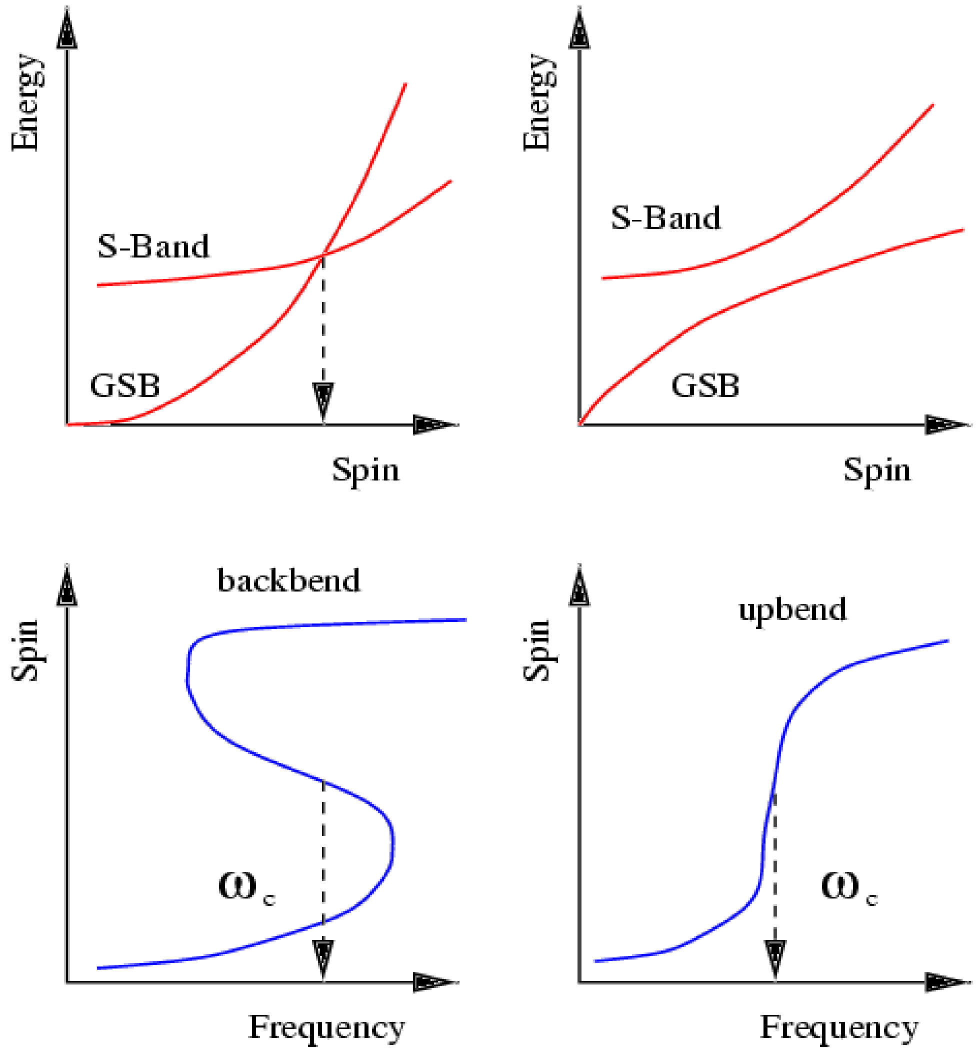


- The moment of inertia increases with increasing rotational frequency
- Around spin $10\hbar$ a dramatic rise occurs
- The characteristic 'S' shape is called a backbend (^{158}Er)
- A more gradual increase is called an upbend (^{174}Hf)

Backbending Movie



Band Crossings



- Backbending can be interpreted as the crossing of two bands
- The 'G' band (Ground state) is a fully paired configuration
- The 'S' band (Super or Stockholm) contains one broken pair

Quadrupole Pairing

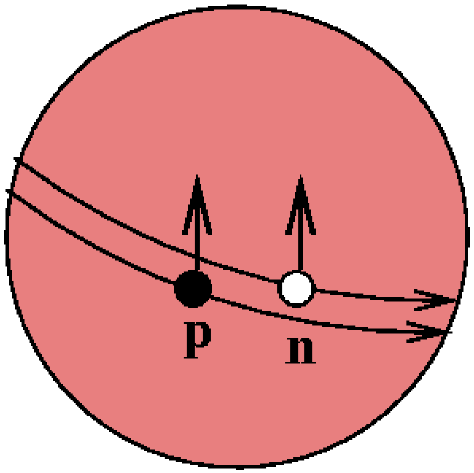
- Higher order pairing correlations may occur leading to configuration-dependent pairing which depends on the relative orientation of nuclei orbits in a deformed potential

- The Y_{21} quadrupole component has the largest effect on the moment of inertia. The nuclear shape still has Y_{20} symmetry and hence the quadrupole pairing is of a dynamical nature

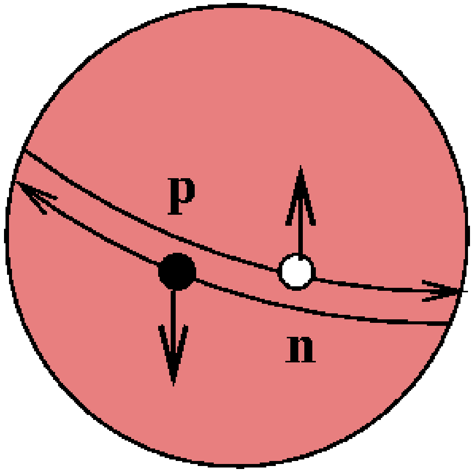
- The generalised pair creation operator is:

$$P_{\lambda\mu}^{\dagger} = \sum \langle u_1 | r^{\lambda} Y_{\lambda\mu} | u_2 \rangle \; a_{u1}^{\dagger} \, a_{\bar{u}2}^{\dagger}$$

Neutron-Proton Pairing



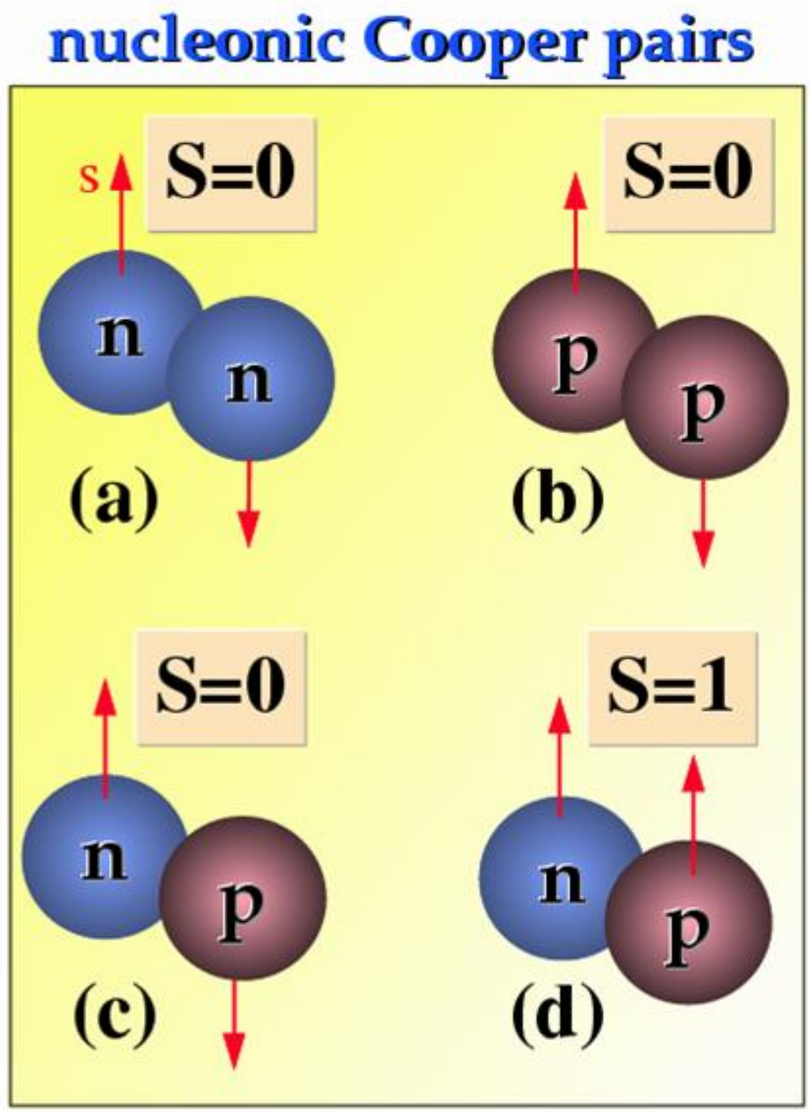
- The concept of superconductivity, related to like nucleon pairs coupled to spin I = 0 and isospin T = 1, can be extended to neutron-proton pairs with T = 0



- The greatest overlap occurs if the particles are in the same orbitals

- Strong neutron-proton pairing can occur for nuclei with N = Z

Nucleon Pairing



- The isovector (T=1) n-p pairing (c) is similar to the n-n (a) and p-p (b) pairing
- The isoscalar (T=0) n-p pairing (d) is clearly different